1 Assigment and final tasks.

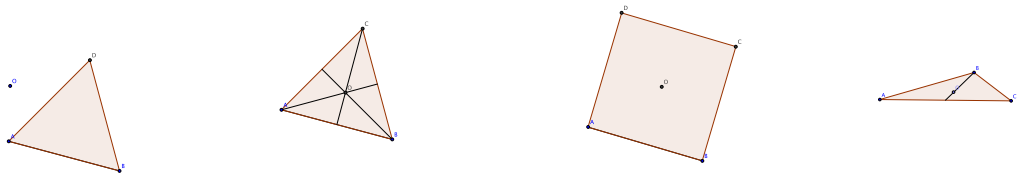
The mathematical task is to tell reasons why any decorative art piece looks beautiful in church architecture ? You will explain how this has to do with symmetries hidden in the motif.

The communication task is a peer to peer students conference based on a self made poster to be given as experts in direction to visitors. All members of the team have to speak equally when presenting to the visitors.

The collaborative task is a team work in a foreign language and members are called below Alice, Bob, Charlie, David and Emmy. Feel free to share roles.

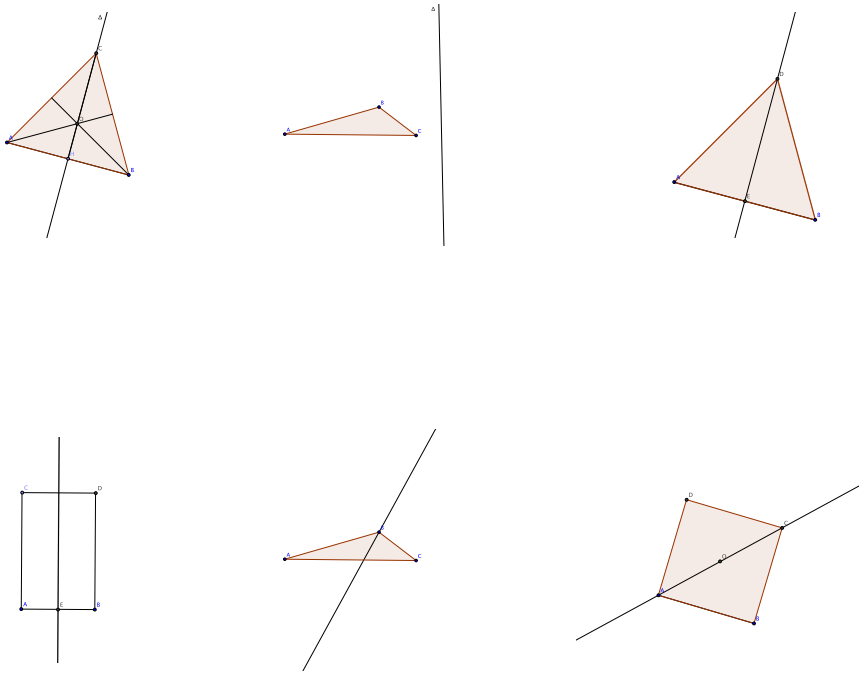
2 Demonstrating geometric functions with the Labosaïque

1. A first generic geometric function : Alice and Bob sketch diagrams for the poster following the instructions and answering the questions below.
   (a) Let O be a point and angle $\dfrac{360^\circ}{\mathbf{2}} = 180^\circ$. Consider rotation about center point O by angle 180°. Take a point M and rotate about O by angle 180° to obtain point M'. This function giving point M' starting with point M is the rotation denoted by r_2. This function is also known as a symmetry of center point O.
   (b) Choose a shape suggested in the given documents (scalene triangle, equilateral triangle, square) and rotate it under r_2. Is this image shape the same as the starting shape ? Do both shapes coincide exactly on the same spot ? Explain why or why not.
   (c) Can you find other shapes whose image under r_2 coincides exactly with the starting shape ?



2. A second generic geometric function : Charlie sketches diagrams for the poster following the instructions below.
   (a) Let d be a straight line and consider the reflection of axis d. *For further reasons this line is taken passing through point* O. Take any point N and reflect it about d to obtain a new point N'. This function giving point N' starting with point N is the reflection denoted by s_d.
   (b) Choose a shape suggested in the given documents (scalene triangle, equilateral triangle, square) and reflect it under s_d. Is this image shape the same as the starting shape ? Do both shapes coincide exactly on the same spot ? Explain why or why not.

(c) Can you find other shapes whose image under s_d coincides exactly with the starting shape ?



3. A third generic geometric function : David and Emmy sketch diagrams for the poster following the instructions below.

   (a) Consider one move horizontally, from left to right, of given length 3 units, for example. This move instruction is showed in mathematics as an arrow, called vector $\overrightarrow{u}$, of length 3, horizontal direction and oriented from left to right. Take any point N and translate it following vector $\overrightarrow{u}$ to obtain a new point N'. This function giving point N' starting with point N is the translation denoted by $t_{\overrightarrow{u}}$.

   (b) Choose a shape suggested in the given documents (scalene triangle, equilateral triangle, square) and translate it under $t_{\overrightarrow{u}}$. Is this image shape the same as the starting shape ? Do both shapes coincide exactly on the same spot ? Explain why or why not.

   (c) Can you find any shape whose image under $t_{\overrightarrow{u}}$ coincide exactly with the starting shape ?

4. In the following, all team members combine the effects of these functions r_2, s_d and $t_{\overrightarrow{u}}$ on one point and draw the corresponding diagrams. Straight line d and vector $\overrightarrow{u}$ are taken below with same direction.

   (a) Consider one point M (or shape F), apply reflection s_d giving new point M' and $t_{\overrightarrow{u}}$ applied next to M' giving new point M'' (or new shape F'').

   (b) Consider the new function starting with point M and obtaining point M'' as an image of M. This new function is denoted by $t_{\overrightarrow{u}} s_d$ in this order : it is called a glide reflection of axis d and vector $\overrightarrow{u}$.

   (c) Repeat the process given in the preceding question for function $s_d t_{\overrightarrow{u}}$. Be careful to write letters in the correct order.

   (d) Have the glide reflections $t_{\overrightarrow{u}} s_d$ and $s_d t_{\overrightarrow{u}}$ the same effect on any point M ?

5. Alice and Bob demonstrate these geometric functions on shapes using the Labosaïque equipment to allow visitors to understand their findings above. Ask the teachers for help to use the equipment.

3 Globally invariant pattern under a geometric function

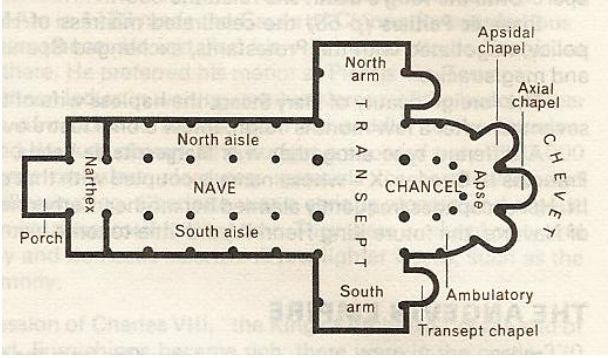
A given shape is called **globally invariant under the geometric function f** if its image under f stays the same.

1. An equilateral triangle is globally invariant under any reflection whose axis is also a perpendicular bisector of any triangle side. Alice draws a diagram to show this property. Bob draws another sort of triangle and experiences the effect of the same function to decided if his triangle stays globally invariant or not.
2. A square is globally invariant under any reflection whose axis is also a perpendicular bisector of the square side. Is there any other possible reflection axis leaving the square globally invariant ? David draws a diagram to show this property. Charlie draws a rectangle and experiences the effect of the same function to decide if his rectangle stays globally invariant or not.
3. Consider now a frieze and note that it is always globally invariant under a translation chosen with appropriate vector.
4. For some particular frieze, note that it is sometimes only globally invariant under a glide reflection chosen with appropriate vector and axis of reflection.

Now, we apply these mathematical ideas to art and architecture.

4 Looking for a geometric pattern in architecture

1. Below is the diagram of a church. Each member of the team has to locate on the plan where the picture of a motif has been taken in the church.






2. For each decorative pattern above show under which geometric function it stays globally invariant. Explain.
3. For each function used above draw clearly all its elements on the given photos.
4. Look for other geometric patterns in this architectural site of the Ladies Abbey church and comment on their mathematical insight.

5 Provided material

the Labosaïque equipment
a measure tape
a combined protractor square set ruler (a "Math en Main" present to each participant)
copies of geometric drawings (scalene triangle, equilateral triangle, square) to be completed
blank sheets for further drawings to be glued onto the poster
separate photos of motifs in the church and the church plan